Overview

1. What is “super-resolution”
   a. Diffraction
   b. STORM

2. Compressed Sensing
   a. Applied to STORM

3. Light Sheet Imaging
   a. Lattice-Light Sheets
Natural Resolution Limits: Diffraction

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Natural Resolution Limits: Diffraction

For typical cameras

\[ d = 1.22 \times \lambda \times f\# \]

Raleigh Criterion

For microscopes

\[ d = \frac{\lambda}{2n \sin \theta} = \frac{\lambda}{2 \times NA} \]

Abbe diffraction limit

NA is typically 0.1-0.4 for common lenses in air, up to 1.0-1.5 for oil lenses.

iPhone 7:
\[ = 1.22 \times 650\text{nm} \times f/1.8 \]
\[ = 1.4 \text{ \mu m} \]
pixels are only 1.22 \text{ \mu m}!

Typical Limit:
\[ = \frac{500\text{nm}}{(2 \times 1.25)} \]
\[ = 0.2 \text{ \mu m} = 200\text{nm} \]
Microtubules are \(~24\text{nm}\)
STORM: Stochastic Optical Reconstruction Microscopy

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Compressed Sensing (a.k.a. Sparse Sampling)

If your data is “compressible”, you can take just a handful of random measurements, and, using “simple” math, you can reconstruct your data (with minimal error and high probability)


$$\min \|x\|_{\ell_1} \text{ subject to } \|Ax - y\|_{\ell_2} \leq \epsilon.$$
Compressed Sensing (a.k.a. Sparse Sampling)

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Compressed Sensing (a.k.a. Sparse Sampling)

Davenport, Duarte, Eldar, Kutynoik, *Introduction to Compressed Sensing*
Compressed Sensing

Compressed Sensing

Real Picture (65,536 pixels)

CS Reconstruction (3,300 samples)

CS Reconstruction (1,300 samples)

CS Reconstruction (6,500 samples)
Faster STORM using compressed sensing


1. Acquire PSF
2. Get Image
3. Increase Grid
4. Solve CS problem
Faster STORM using compressed sensing


Denser Images!  Many times denser  More precise  Faster imaging
Faster STORM using compressed sensing
Faster STORM using compressed sensing

40% pixels on

40% on, CS Solve

CS
50 readings
4% Density

Classic
1000 readings
~0.8% Density

https://github.com/leonidk/cs371
Quantitative Comparison


https://github.com/hrouault/Brecs
Extra Slides
Faster STORM using compressed sensing

Solve

\[ w_0(\cdot) + \ldots + w_{205}(\cdot) + \ldots + w_{819}(\cdot) + \ldots = \]

With

\[ \min \| w \|_1 \]

Gives

\[ w \in \mathbb{R}^{1024} \Rightarrow w \in \mathbb{R}^{32 \times 32} \]
DAOSTORM

FALCON

Figure S1. The first-order switching kinetics of the molecular switch. A, The number of molecules remaining fluorescent as a function of time after the green laser was turned off. A single exponential fit of the data (solid line) gives $k_{\text{off}} = 0.4 \text{ s}^{-1}$. B, The number of molecules that were converted back to the fluorescent state as a function of time after the green laser was turned on. A single exponential fit (solid line) gives the observed rate constant for switching Cy5 on ($k_{\text{on, obs}} = 1.1 \text{ s}^{-1}$). Considering the competing actions of the red and green lasers, the actual rate constant $k_{\text{on}}$ for switching the dye on by the green laser is equal to $k_{\text{on, obs}} - k_{\text{off}}$. Data in A and B are not from the same experiment.
Abstract

Suppose we are given a vector $f$ in a class $\mathcal{F} \subset \mathbb{R}^N$, e.g. a class of digital signals or digital images. How many linear measurements do we need to make about $f$ to be able to recover $f$ to within precision $\epsilon$ in the Euclidean ($\ell_2$) metric?

This paper shows that if the objects of interest are sparse in a fixed basis or compressible, then it is possible to reconstruct $f$ to within very high accuracy from a small number of random measurements by solving a simple linear program. More precisely, suppose that the $n$th largest entry of the vector $|f|$ (or of its coefficients in a fixed basis) obeys $|f|_{(n)} \leq R \cdot n^{-1/p}$, where $R > 0$ and $p > 0$. Suppose that we take measurements $y_k = \langle f, X_k \rangle$, $k = 1, \ldots, K$, where the $X_k$ are $N$-dimensional Gaussian vectors with independent standard normal entries. Then for each $f$ obeying the decay estimate above for some $0 < p < 1$ and with overwhelming probability, our reconstruction $f^\sharp$, defined as the solution to the constraints $y_k = \langle f^\sharp, X_k \rangle$ with minimal $\ell_1$ norm, obeys

$$\|f - f^\sharp\|_{\ell_2} \leq C_p \cdot R \cdot (K / \log N)^{-r}, \quad r = 1/p - 1/2.$$  

There is a sense in which this result is optimal; it is generally impossible to obtain a higher accuracy from any set of $K$ measurements whatsoever. The methodology extends to various other random measurement ensembles; for example, we show that similar results hold if one observes few randomly sampled Fourier coefficients of $f$. In fact, the results are quite general and require only two hypotheses on the measurement ensemble which are detailed.
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Superresolution fluorescence microscopy

Leonid Keselman, Daniel Fernandes
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&= 1.4 \text{µm}
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![Image of denser images comparison](image)
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w =
**FALCON**

Supplementary Figure: Switching kinetics

**Figure S1.** The first-order switching kinetics of the molecular switch. **A,** The number of molecules remaining fluorescent as a function of time after the green laser was turned off. A single exponential fit of the data (solid line) gives $k_{\text{off}} = 0.4 \text{ s}^{-1}$. **B,** The number of molecules that were converted back to the fluorescent state as a function of time after the green laser was turned on. A single exponential fit (solid line) gives the observed rate constant for switching Cy5 on ($k_{\text{on,obs}} = 1.1 \text{ s}^{-1}$). Considering the competing actions of the red and green lasers, the actual rate constant $k_{\text{on}}$ for switching the dye on by the green laser is equal to $k_{\text{on,obs}} - k_{\text{off}}$. Data in **A** and **B** are not from the same experiment.
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